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Maged Elshamy, Ph.D.

Alabama A&M University, Huntsville, AL

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January 2002

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Maged Elshamy, Ph.D.  
Alabama A&M University, Huntsville, AL

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## **1. Introduction**

Humans are involved in every man-machine system. More often accidents and systems failure are traced to human errors. As such humans play an important role in the overall reliability of engineering systems because various systems are interconnected by human links. Therefore, in order to have meaningful system reliability analysis, the reliability of the human element must be taken into consideration.

Human reliability can be improved significantly by following human-factor principles during the system design phase. Additional enhancement may be obtained from careful selection and training of appropriate personnel. Appropriate design of the system and equipment also help to increase human reliability. However, even when selection and training are efficiently carried out and appropriate design features are incorporated, people are not always reliable. They make mistakes, and in some cases, their errors will lead to systems failure and accidents.

In this research effort, the issue of human errors in the processing of the shuttle and other flight programs is investigated. Such events as sending the wrong command or assembling the wrong parts tend to recur despite efforts at prevention. In studying human errors, one of the important problems is how to represent the human variables quantitatively in mathematical models. Therefore, stochastic models to address the occurrence of human errors in certain systems are needed. This report, which was completed and submitted to KSC on June 23, 1999, presents such stochastic models. These models have never been used in this particular application before.

The availability of probabilistic models of the processing work flow might offer the opportunity to analyze contributing factors and test process improvements that could reduce or eliminate these events. These models may enable us to predict trigger events

that lead to failure modes and affect human performance. The availability of technologies to improve situational awareness will increase systems safety.

A human error is defined as a failure to perform a prescribed task or the performance of a prohibited action that could result in damage to equipment and property or disruption of scheduled operations. Various factors may lead to a human error. Some of these factors are inadequate lighting, high noise level, improper tools, stress, fatigue, and poor equipment design.

In order to prevent errors, we must first be able to predict them. Therefore, the objective of this research effort is to establish probabilistic models substantiated by sound theoretic foundation to address the occurrence of human errors in the processing of the space shuttle and other flight programs.

## **2. Human Variables**

Human errors may also be caused by human variables such as proficiency, fatigue, learning, stress reaction, group interaction variables, norms and goals, group identification, cohesiveness, social pressure, and morale. This topic is covered extensively in the open literature, see [2] and the reference therein, here we shed the light on two of such variables as examples,

**(A) Fatigue:** Fatigue is one of the principal human variables which lead to errors. A definition of fatigue is the subjective residue of feelings of bodily discomfort and aversion to effort. Fatigue may be caused by long work shifts, insufficient rest, monotony (tedious sameness), surroundings (poor lighting, noise, atmospheric conditions), mental factors



(responsibility, worries, conflicts), illness and pain, eating habits, etc. It is generally considered a time-correlated disorder that affects performance in many ways such as:

- (a) Responses are made too late or too early,
- (b) responses are made too intensely,
- (c) occasional response omissions are typical, and
- (d) increased fatigue leads to decreased performance, and mean performance time decreases.

For example, in the case of commercial airline pilots, the frequency of human error (e.g., forgetfulness, improper adjustment of controls, etc.) will increase if a pilot is fatigued.

Acute fatigue is generally easier to identify and relieve. Chronic fatigue is a generalized response to stress over a period of time. Thus, cumulative effects of the work situation become important.

**(B) Stress:** This is another important human variable that affect human performance and its reliability. There is a nonlinear relationship between human performance and stress: when the stress is moderate the performance is highest. Otherwise, at very low stress, the task will be unchallenging and dull and thereby human performance will not be at its peak. On the other hand, stress above a moderate level will cause human performance to decline. Reasons for the decline include, for example, worry, fear or other kind of psychological stress. Moderate stress may be defined as the level of stress enough to keep the human being alert.

### **3. Characteristics of Human Error**

There are some similarities between man (with multiple organs and functions) and a machine (with multiple components and functions) in terms of their proneness to failure, which leads to parallelism of methods of analysis in each. However, the human failure process has its peculiar features. Human errors are of random recurring type, whereas hardware failure condition is irreversible by itself. That is why hardware reliability is concerned with the first failure. A human may continually improve his performance from learning unlike his machine-counterpart. There are three important aspects of human error:

- a) the possibility of self-detection or detection by another person,
- b) the ability to be corrected or recovered, and
- c) its consequences.

When an error is made, it may or may not be recognized by the operator as an error. If it is not recognized, then the operator cannot take any further action. If it is recognized, it may or may not be correctable. In most cases, operators are motivated to take action to correct errors if they are able. However, if the error is not correctable, the consequences must be considered. Such consequences may be major or minor. Major consequences would require the operator to take immediate alternative action, but minor ones may only require continued monitoring to see if further action is required at a later time.

#### 4. Learning Curves

A graphic profile of the learner's performance reflecting an increase of the speed and/or accuracy is called a learning curve. Such a curve plots some measure of performance (such as speed, accuracy, errors, hits, etc.) against some measure of the amount of practice (e.g. trials, days). Individuals can learn new working skills; groups can learn cooperative skills. The increased productivity resulting from learning a certain operation can influence the completion time of a task. The relationship between completion time and number of tasks completed is expressed by an exponential function

$$T = a n^{-b} \quad (1)$$

where,

$n$  = number of tasks accomplished

$a$  = time required to complete the first task

$b$  = positive exponent associated with the learning rate

$T$  = the accumulated average time per task

Each time the operation is repeated, the curve forecasts decrease in time. The decrease in time is the result of improved methods, procedures, or worker familiarization.

Remark: If  $b = 0$  no learning occurs. In case of forgetfulness or inattention,  $b < 0$ , and  $T$  increases in time.

#### **Definition: The Learning Rate**

The reduction ratio of the accumulated average times in doubling the number of tasks accomplished is defined as the learning rate  $r$ ,

$$r = \frac{a(2n)^{-b}}{a n^{-b}} = 2^{-b} \quad (2)$$

The learning rate remains relatively constant for a given individual, group, or industry.

Therefore,

$$b = - \frac{\ln r}{\ln 2} \quad (3)$$

In practical situations,  $r$  varies from 50% (fast learning) to 100% (no measurable learning). If forgetting occurs as bad habits are picked up, a theoretical learning rate could exceed the 100% limit.

## **5. Modes of Error Detection**

People's errors are brought to their attention in three ways:

- a) directly, they can find out for themselves through various kinds of self-monitoring,
- b) environmental indication that makes it very clear that they made an error, and
- c) indirectly, the error may be discovered by another person, who then tells them.

## **6. Stochastic Models of Human Errors**

Probabilistic models of human performance in discrete and continuous tasks are developed in this section. The approach used in these models is to view the human as a whole (as a black box) with a finite number of unknown parameters. Once the structural model of the operator is chosen, the problem is reduced to one of parameter estimation.

## Model 1

### Discrete Model

Nonstationary Bernoulli Trials: These are Bernoulli trials with variable probabilities from trial to trial. In a sequence of a repetitive trials of a given task, a human can fail to perform a prescribed act or perform a prohibited act, thus causing a system failure. Here we assume that the probability of committing an error decreases as the number of trials increases in accordance to the learning curve theory. That is, the operator continually improves his performance from learning.

Let  $\{X_1, X_2, X_3, \dots, X_n\}$  be a finite sequence of mutually independent random variables taking the values 0 (No error), or 1 otherwise. Assume that the error probabilities are independent of the past performance record (previous tasks).

Let  $p_k = P(X_k = 0)$ ,  $q_k = P(X_k = 1) = 1 - p_k$ , and

$$\begin{aligned} S_n &= \text{the total number of errors in the first } n \text{ independent trials} \\ &= \sum_{k=1}^n X_k \end{aligned}$$

The expected value of  $S_n$  is

$$M_n = E[S_n] = \sum_{k=1}^n E[X_k] = \sum_{k=1}^n q_k$$

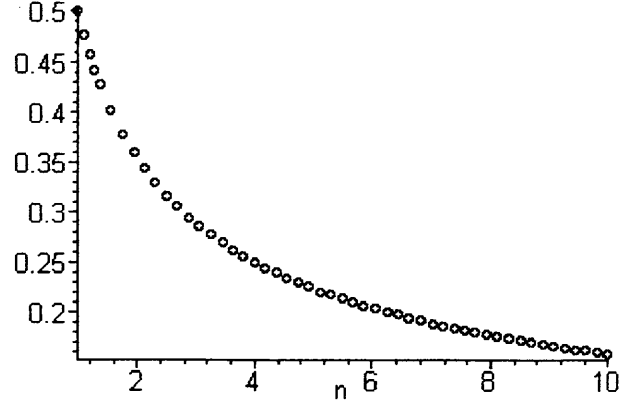
The average probability of an error in  $n$  trials is

$$q = \frac{\sum_{k=1}^n E[X_k]}{n} = \frac{M_n}{n} \quad (4)$$

A model shaped after the general learning curve can be constructed by setting the average probability of an error, Equation (4), equal to an exponential function in Equation similar to that in Equation (1), see [5], to obtain

$$\frac{M_n}{n} = a n^{-b} \quad (5)$$

where  $a > 0$  and  $b > 0$  are parameters to be estimated. The parameter  $a$ ,  $0 \leq a \leq 1$ , represents the error probability on the first trial ( $n = 1$ ).



A graph of the average probability of committing an error  $\frac{M_n}{n} = a n^{-b}$

An expression for the error probability on the  $n$ th trial can be obtained from Equation (4),

$$\begin{aligned} q_n &= \sum_{k=1}^n q_k - \sum_{k=1}^{n-1} q_k \\ &= a[n^{1-b} - (n-1)^{1-b}] \end{aligned} \quad (6)$$

The total expected number of errors in a fixed sequence of trials from trial number  $n_1$  to trial  $n_2$  is

$$M_{n_2} - M_{n_1-1} = a[n_2^{1-b} - (n_1-1)^{1-b}] \quad (7)$$

The human reliability that a prescribed sequence of successive trials from trial number  $n_1$  through trial number  $n_2$  is completed without errors is

$$R(n_1, n_2) = \prod_{k=n_1}^{n_2} p_k = \prod_{k=n_1}^{n_2} (1 - q_k) \quad (8)$$

**Parameter Estimation:**

Let  $m_1$  = the number of errors occurred in the first  $n_1$  trials

$m_2$  = the number of errors occurred in the sequence of trials from trial number  $(n_1 + 1)$  through trial number  $n_2$

Then,

$$m_1 = a n_1^{1-b} \quad (9)$$

$$m_2 = a n_2^{1-b} - m_1 = a n_2^{1-b} - a n_1^{1-b} \quad (10)$$

and

$$\frac{m_1}{m_2} = \frac{n_1^{1-b}}{n_2^{1-b} - n_1^{1-b}}$$

$$m_2 n_1^{1-b} = m_1 (n_2^{1-b} - n_1^{1-b})$$

$$\frac{m_1}{m_1 + m_2} = \left( \frac{n_1}{n_2} \right)^{1-b}$$

or

$$\ln\left(\frac{m_1}{m_1 + m_2}\right) = (1 - b) \ln\left(\frac{n_1}{n_2}\right)$$

From which

$$\hat{b} = 1 - \left[ \frac{\ln m_1 - \ln(m_1 + m_2)}{\ln n_1 - \ln n_2} \right] \quad (11)$$

Now Equation (10), yields

$$m_1 + m_2 = a n_2^{1-b}$$

or

$$\hat{a} = (m_1 + m_2) n_2^{\hat{b} - 1} \quad (12)$$

Remarks: 1. If there is no learning  $r = 100\%$  and, from equation (2),  $r = 2^{-b}$ , so  $b = 0$ . Therefore, from Equation (6),  $q_1 = q_2 = q_3 = \dots = a$ , and the process degenerates into an ordinary Bernoulli trials (as expected).

2. For large values of  $n$  and moderate values of  $M_n = \sum_{k=1}^n q_k$ ,  $M_n := \lambda$  for large  $n$ , it can be shown that  $S_n$ , the distribution of the total number of errors in the first  $n$  independent trials is approximately Poisson, see [3].

$P(S_n = k) = \frac{\lambda e^{-\lambda}}{k!}$ , where an estimate of  $\lambda$  (using Equation (5)) is

$$\hat{\lambda} = \hat{a} n^{1-b}.$$



## Model 2

### Discrete Model

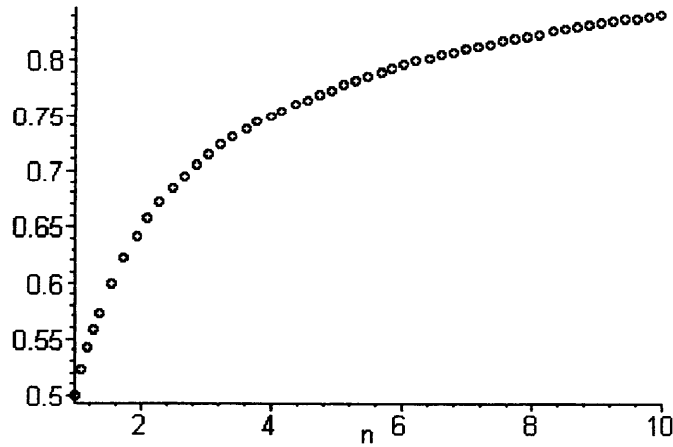
As in model 1, we assume Bernoulli trials with variable probabilities from trial to trial. Here we assume that the probability of committing an error increases as the number of trials increases exponentially because the operator's performance is deteriorating due to, say, stress or fatigue. Since  $\frac{M_n}{n}$  is an average probability, it should be nonnegative and bounded above by 1, and in this case nondecreasing, so instead of Equation (5) we use

$$\frac{M_n}{n} = 1 - a n^{-b}$$

or

$$M_n = n - a n^{1-b} \quad (13)$$

where  $a > 0$  and  $b > 0$  are parameters to be estimated. The quantity  $(1 - a)$ ,  $0 \leq a \leq 1$ , represents the error probability on the first trial ( $n = 1$ ).



A graph of an increasing (average) probability of committing an error  $\frac{M_n}{n} = 1 - a n^{-b}$

In this case an expression for the error probability on the  $n$ th trial can be obtained from Equation (13),

$$\begin{aligned}
 q_n &= \sum_{k=1}^n q_k - \sum_{k=1}^{n-1} q_k \\
 &= n - an^{1-b} - [n-1 - a(n-1)^{1-b}] \\
 &= 1 - a[n^{1-b} - (n-1)^{1-b}]
 \end{aligned} \tag{14}$$

The total expected number of errors in a fixed sequence of trials from trial number  $n_1$  to trial number  $n_2$  is

$$M_{n_2} - M_{n_1-1} = n_2 - n_1 - a(n_2^{1-b} - n_1^{1-b}) \tag{15}$$

The human reliability that a prescribed sequence of successive trials from trial number  $n_1$  through trial number  $n_2$  is completed without errors is

$$R(n_1, n_2) = \prod_{k=n_1}^{n_2} p_k = \prod_{k=n_1}^{n_2} (1 - q_k) \tag{16}$$

### Parameter Estimation:

Let  $M_1$  = the number of errors occurred in the first  $n_1$  trials

$M_2$  = the number of errors occurred in the sequence of trials from trial number  $(n_1 + 1)$  through trial number  $n_2$

Then,

$$M_1 = n_1 - a n_1^{1-b} \tag{17}$$

$$M_1 + M_2 = n_2 - a n_2^{1-b} \tag{18}$$

and

$$\frac{n_1 - M_1}{n_2 - M_1 - M_2} = \frac{n_1^{1-b}}{n_2^{1-b}} = \left(\frac{n_1}{n_2}\right)^{1-b}$$

or

$$\ln\left(\frac{n_1 - M_1}{n_2 - M_1 - M_2}\right) = (1 - b) \ln\left(\frac{n_1}{n_2}\right)$$

From which

$$\hat{b} = 1 - \left[ \frac{\ln(n_1 - M_1) - \ln(n_2 - M_1 - M_2)}{\ln n_1 - \ln n_2} \right] \quad (19)$$

Now Equation (18), yields

$$n_2 - M_1 - M_2 = a n_2^{1-b}$$

or

$$\hat{a} = (n_2 - M_1 - M_2) n_2^{\hat{b} - 1} \quad (20)$$

Remark: If the average error probability  $\frac{M_n}{n}$  does not increase then, from its definition,  $b = 0$ . Therefore, from Equation (14),  $q_1 = q_2 = q_3 = \dots = (1 - a)$ , and the process degenerates into an ordinary Bernoulli trials (as expected).

### Model 3

#### Continuous Time Model (Decreasing Mean Value Function)

The system is being improved over time or the operator continually improves his performance from learning in continuous time tasks. Learning is assumed to take place in a systematic way compatible with the general learning curve.

Assumptions: 1. Human errors occur at random times, and each error is treated as an event without duration. The number of human errors by time  $t$  is denoted by  $N(t)$ .  
2. The operator continually improves his performance from learning.  
3. The number of human errors during nonoverlapping intervals do not affect each other. This means that the counting process  $\{N(t): t \geq 0\}$  has independent increments.

**Definition:** The counting process  $\{N(t): t \geq 0\}$  is called a nonhomogeneous (or non stationary) Poisson process, see [6], with intensity function  $\lambda(t)$ ,  $t \geq 0$ , if

- (a)  $N(0) = 0$
- (b)  $\{N(t): t \geq 0\}$  has independent increments,
- (c) In any time interval (no matter how small) there is a positive probability that an event will occur (but is not certain), i.e., for any  $t > 0$ ,  $0 < P(N(t) > 0) < 1$ .
- (d) In sufficiently small intervals, at most one event can occur, i.e., it is not possible for events to happen simultaneously.

Let  $M(t) = E[N(t)] = \text{Mean Value function}$

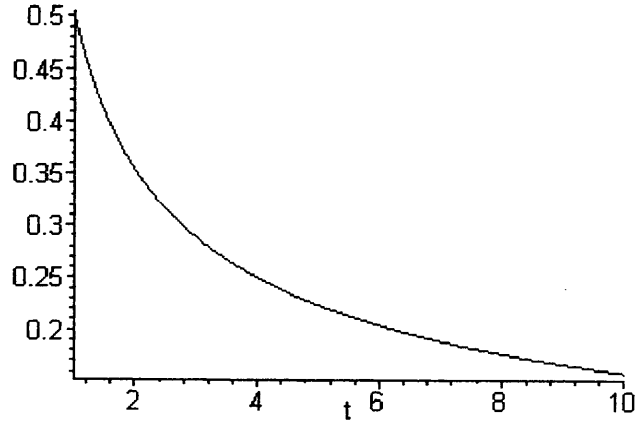
The intensity function  $\lambda(t)$  is the rate of change of the mean value function  $M(t)$ , i.e.,  $\lambda(t) = \frac{M(t)}{dt}$  or  $M(t) = \int_0^t \lambda(s) ds$ . The distribution of the number of “error events” in the an interval  $(t, t + s]$  is given by

$$P(N(t + s) - N(t) = j) = e^{-(M(t+s) - M(t))} \frac{(M(t+s) - M(t))^j}{j!}, \quad j \geq 0 \quad (21)$$

To construct a model fashioned after the general learning curve theory, see [5], assume that the time-averaged mean value function takes the form

$$\frac{M(t)}{t} = \lambda t^{-\alpha}, \quad t > 0, \quad \alpha > 0 \quad (22)$$

The parameters  $\lambda \geq 0$  and  $\alpha \geq 0$  are to be estimated. The parameter  $\lambda$  represents the mean value (expected number) of errors at the first unit time,  $t = 1$ .



A graph of time-average mean value function  $\frac{M(t)}{t} = \lambda t^{-\alpha}$

The counting process  $\{N(t) : t \geq 0\}$  is a nonhomogeneous Poisson process (or a Poisson process with nonstationary increments) with mean value function (or expectation function)

$$M(t) = \lambda t^{1-\alpha} \quad (23)$$

Therefore,

$$P\{N(t_2) - N(t_1) = k\} = \frac{(\Delta M(t_1, t_2))^k}{k!} e^{-\Delta M(t_1, t_2)}, \quad (24)$$

for any  $0 \leq t_1 \leq t_2$ ,  $k \geq 0$ ,

where

$$\begin{aligned} \Delta M(t_1, t_2) &= M(t_2) - M(t_1) \\ &= \lambda (t_2^{1-\alpha} - t_1^{1-\alpha}) \end{aligned} \quad (25)$$

The human reliability that a given task of specified time duration  $[t_1, t_2]$  is performed successfully without any errors is

$$R(t_1, t_2) = P\{N(t_2) - N(t_1) = 0\} = e^{-\lambda(t_2^{1-\alpha} - t_1^{1-\alpha})} \quad (26)$$

**Remark:** When there is no learning  $\alpha = 0$ ,  $M(t) = \lambda t$ , and the counting process  $\{N(t) : t \geq 0\}$  degenerates into homogeneous Poisson process (with stationary increments). In this case, the human reliability for the time interval  $[0, t]$  reduces to

$$R(0, t) = e^{-\lambda t} \quad (27)$$

### Parameter Estimation:

Let  $m_1$  = Number of errors occurred in the time interval  $[0, t_1]$

$m_2$  = Number of errors occurred in the time interval  $[t_1, t_2]$

Then

$$m_1 = \lambda t_1^{1-\alpha} \quad (28)$$

$$m_2 = \lambda t_2^{1-\alpha} - m_1 \quad (29)$$

Solving simultaneously, we obtain:

$$\hat{\alpha} = 1 - \frac{\ln(\frac{m_1}{m_1 + m_2})}{\ln(\frac{t_1}{t_2})} \quad (30)$$

or

$$\hat{\alpha} = 1 - \frac{\ln m_1 - \ln(m_1 + m_2)}{\ln t_1 - \ln t_2}$$

and

$$\hat{\lambda} = (m_1 + m_2)(t_2^{\hat{\alpha}-1}) \quad (31)$$

## Model 4

### Continuous Time Model (Increasing Mean Value Function)

Here the operator's performance is continually deteriorating due to certain factor such as long work shifts, insufficient rest, monotony (tedious sameness), surroundings (poor lighting, noise, atmospheric conditions), mental factors (responsibility, worries, conflicts), illness and pain, eating habits, etc.

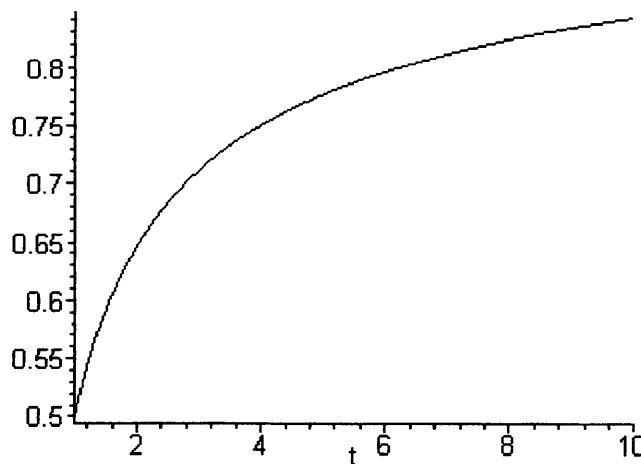
Let

$N(t)$  = Number of errors by time  $t$

The counting process  $\{N(t) : t \geq 0\}$  is a nonhomogeneous Poisson process (or a Poisson process with nonstationary increments). In order to construct a probabilistic model of human errors which accounts for deteriorating performance, the time-average mean value function is assumed to be exponentially increasing of the form

$$\frac{M(t)}{t} = 1 - \lambda t^{-\alpha}, \quad t > 0, \alpha > 0, \quad (32)$$

The parameters  $\lambda \geq 0, \alpha \geq 0$ , are to be estimated. The quantity  $(1 - \lambda)$  represents the mean value (or expected number) of errors committed by time  $t = 1$ .



A graph of time-average mean value function  $\frac{M(t)}{t} = 1 - \lambda t^{-\alpha}$

The counting process  $\{N(t): t \geq 0\}$  is a nondecreasing Poisson process (or a Poisson process with nonstationary increments) with mean value function

$$M(t) = t - \lambda t^{1-\alpha} \quad (33)$$

Therefore,

$$P\{N(t_2) - N(t_1) = j\} = \frac{(\Delta M(t_1, t_2))^j}{j!} e^{-\Delta M(t_1, t_2)}, \quad (34)$$

for any  $0 \leq t_1 \leq t_2, j \geq 0$ ,

where

$$\begin{aligned} \Delta M(t_1, t_2) &= M(t_2) - M(t_1) \\ &= (t_2 - t_1) - \lambda (t_2^{1-\alpha} - t_1^{1-\alpha}) \end{aligned} \quad (35)$$

The human reliability, which is the probability that a given task of specified time duration  $[t_1, t_2]$  is performed successfully without any errors, is therefore

$$R(t_1, t_2) = P\{N(t_2) - N(t_1) = 0\} = e^{-(t_2 - t_1) - \lambda (t_2^{1-\alpha} - t_1^{1-\alpha})} \quad (36)$$

Remark: If  $\alpha = 0$  (no cause of deteriorating performance),  $M(t) = ct$ , where  $c = 1 - \lambda$  is a constant. In this case the counting process  $\{N(t): t \geq 0\}$  degenerates into homogeneous Poisson process (with stationary increments), and the human reliability for the time interval  $[0, t]$  reduces to

$$R(0, t) = e^{-ct} \quad (37)$$



**Parameter Estimation:**

Let  $m_1$  = Number of errors occurred in the time interval  $[0, t_1]$

$m_2$  = Number of errors occurred in the time interval  $[t_1, t_2]$

Then

$$m_1 = t_1 - \lambda t_1^{1-\alpha} \quad (38)$$

$$m_2 = t_2 - \lambda t_2^{1-\alpha} - m_1 \quad (39)$$

Solving simultaneously as before we obtain:

$$\hat{\alpha} = \frac{\ln\left[\frac{t_1 - m_1}{t_2 - (m_1 + m_2)}\right]}{\ln\left(\frac{t_1}{t_2}\right)} - 1$$

or (40)

$$\hat{\alpha} = 1 - \frac{\ln(t_1 - m_1) - \ln(t_2 - m_1 - m_2)}{\ln t_1 - \ln t_2}$$

and

$$\hat{\lambda} = (t_2 - m_1 - m_2)t_2^{\hat{\alpha}-1} \quad (41)$$

## Model 5

### Continuous Time Model

#### Hazard Rate Technique

The general human performance reliability function for continuous time tasks can be developed the same way as the case of the classical reliability function. Examples of continuous time tasks performed by humans are aircraft maneuvering, missile countdown, and monitoring. Two methods are presented here, and it will be shown that the two methods lead to the same result. We first recall basic definitions

#### Definition 1: Hazard Rate Function (or Human Error Rate)

Consider a positive continuous random variable  $X(t)$ , e.g. the lifetime of some item or time to failure, having distribution function  $F$  and probability density function  $f$ . The hazard rate (human error rate, or failure rate) function  $h(t)$  of  $F$  is defined by

$$h(t) = \frac{f(t)}{1 - F(t)} \quad (42)$$

**Definition 2: The Human Performance Reliability at Time  $t$ ,  $R(t)$ ,** is the probability of performing a task without errors till time  $t$ . Thus

$$R(t) = P(X > t) = 1 - F(t) \quad (43)$$

The time-dependent human error rate or hazard rate function  $h(t)$  is usually (using Equation (42)) written as

$$h(t) = \frac{-1}{R(t)} \frac{dR(t)}{dt} \quad (44)$$

Integrating both sides over the time interval  $[0, t]$ , we obtain

$$\int_0^t h(s) ds = - \int_0^t \frac{1}{R(s)} dR(s), \text{ where } R(0) = 1. \text{ Thus,}$$

$$R(t) = e^{-\int_0^t h(s) ds} \quad (45)$$

The reliability expression in Equation (45) holds whether the human error rate (or hazard rate) is a constant or nonconstant. We note here that the human error rate could be described by probability distributions such as exponential, Rayleigh, Weibull, normal, or bathtub distributions. For instance, the Weibull density function is known to fit the experimental data in the case of vigilance task, see reference [4].

If we model  $N(t)$  the number of human errors that occur in the time interval  $[0, t]$  as a Markov process, then the following assumptions are appropriate:

- (a)  $N(0) = 0$
- (b) Independent increment assumption: The number of errors that occur in disjoint time intervals are independent.
- (c) Stationary: The distribution of the number of errors that occur in a given interval depends only on the length of that interval and not on its time-location, i.e., the probability distribution of  $N(t + s) - N(s)$  is the same for all  $s \geq 0$  and  $t > 0$ .
- (d)  $P\{N(\Delta t) = 1\} = \lambda \Delta t + o(\Delta t)$
- (e)  $P\{N(\Delta t) \geq 2\} = o(\Delta t)$

Remarks: 1. Assumptions (a) through (e) mean that the process  $\{N(t): t \geq 0\}$  is a Poisson process (a type of Markov process) having intensity parameter  $\lambda$ .

2. For  $s \geq 0$  and  $t > 0$ , the random variable  $N(t + s) - N(s)$ , representing the number of human error over the interval  $(s, s + t]$ , has a Poisson distribution with mean  $\lambda t$ , i.e.,

$$P\{N(t + s) - N(s) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \quad \text{for } k = 0, 1, \dots \quad (46)$$

Let  $P_0(t)$  be the probability that a human is performing a task continuously at time  $t$  without errors, then  $P_0(t) = P\{N(t) = 0\}$  and a differential equation for  $P_0(t)$  can be constructed, using the above assumptions, as follows:

$$\begin{aligned}
P_0(t + \Delta t) &= P\{N(t + \Delta t) = 0\} \\
&= P\{N(t) = 0, N(t + \Delta t) - N(t) = 0\} \\
&= P\{N(t) = 0\}P\{N(t + \Delta t) - N(t) = 0\} \\
&= P_0(t) [1 - P\{N(\Delta t) = 1\} - P\{N(\Delta t) \geq 2\}], \\
&\quad \text{using assumptions (d) and (e),} \\
&= P_0(t) [1 - \lambda \Delta t + o(\Delta t)]
\end{aligned}$$

Thus,

$$\frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} = -\lambda P_0(t) + \frac{o(\Delta t)}{\Delta t}$$

Now, letting  $\Delta t \rightarrow 0$ , we obtain

$$P'_0(t) = -\lambda P_0(t), \text{ where } P_0(0) = 1.$$

Integrating, we obtain

$$P_0(t) = e^{-\lambda t} \tag{47}$$

If the intensity of human error rate  $\lambda$  is not a constant Equation (46) becomes

$$P_0(t) = e^{-\int_0^t \lambda(s) ds} \tag{48}$$

We note that Equation (48) is consistent with Equation (45), since the human reliability function  $R(t) = P_0(t)$ , the probability of committing no errors. Therefore, the two methods lead to the same result. It follows that the probability of committing at least one human error is

$$\left. \begin{aligned}
P_h(t) &= 1 - e^{-\lambda t}, \text{ if } \lambda \text{ is a constant} \\
P_h(t) &= 1 - e^{-\int_0^t \lambda(s) ds}, \text{ if } \lambda \text{ is not a constant}
\end{aligned} \right\} \tag{49}$$

**Example:** Human error rate associated with Weibull distribution.

The Weibull distribution function has the form

$$F(t) = \begin{cases} 0 & t \leq v \\ 1 - \exp\left[-\left(\frac{t-v}{\alpha}\right)^\beta\right] & t > v \end{cases}$$

and the density function is

$$f(t) = \begin{cases} 0 & t \leq v \\ \frac{\beta}{\alpha} \left(\frac{t-v}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{t-v}{\alpha}\right)^\beta\right] & t > v \end{cases}$$

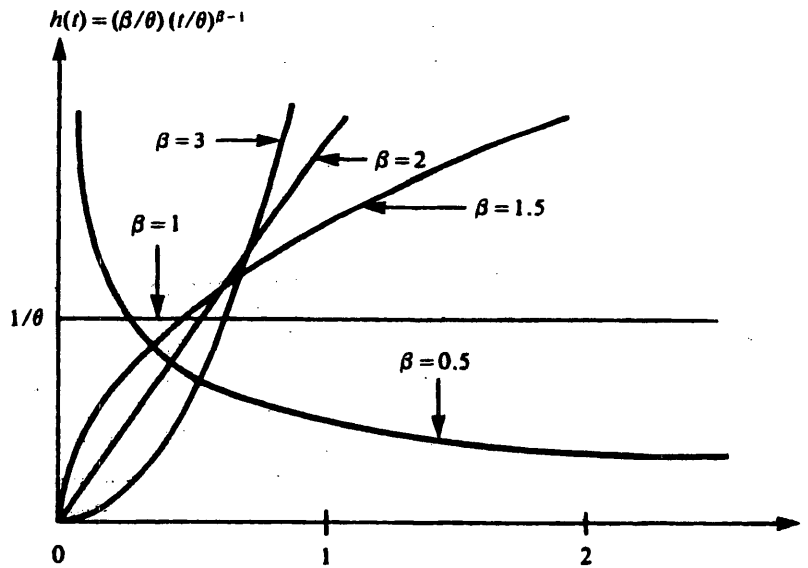
For  $v = 0$ , the time-dependent human error rate (or hazard rate) function, using Equation (42), is

$$h(t) = \frac{\beta}{\alpha} \left(\frac{t-v}{\alpha}\right)^{\beta-1} \Big|_{v=0} = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} \quad (50)$$

and the human performance reliability, using Equation (48), is

$$R(t) = e^{-\int_0^t \frac{\beta}{\alpha} \left(\frac{s}{\alpha}\right)^{\beta-1} ds} = e^{-\left(\frac{t}{\alpha}\right)^\beta}, \quad t > 0 \quad (51)$$

where  $\alpha$  and  $\beta$  are parameters to be estimated using appropriate statistical technique, such as the maximum likelihood estimation method, see [1].



Graphs of the Weibull hazard functions  $h(t)$

Remarks: 1. In the definition of hazard rate function  $h(t)$ , if  $X(t)$  denotes the number of error occurrences in a specified interval  $[0, t]$ , then it can be shown that  $X(t)$  is a nonhomogeneous Poisson process with mean value function  $M(t)$ , where

$$M(t) = \int_0^t h(s) ds.$$

2. Comparing Equations (45) and (48), we see that  $h(t) = \lambda(t)$ . The time-dependent human error rate is the intensity function of the nonhomogeneous Poisson process (see definition in Model 3).

3. An important choice for a nonhomogeneous intensity function is

$$h(t) = \lambda(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1},$$

which gives

$$M(t) = \int_0^t \frac{\beta}{\alpha} \left(\frac{s}{\alpha}\right)^{\beta-1} ds = \left(\frac{t}{\alpha}\right)^{\beta}.$$

In this case the time to first human error occurrence follows a Weibull distribution with parameters  $\alpha$  and  $\beta$ .

4. The intensity parameter  $\lambda(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1}$  is an increasing function of  $t$  if  $\beta > 1$ , a decreasing function of  $t$  if  $\beta < 1$ , and a constant function of  $t$  if  $\beta = 1$ .

5. The  $\beta < 1$  case might apply to a developmental situation in which the system is being improved over time. This is similar to the situation in model 3 where the performance is being improved due to learning.

6. The  $\beta > 1$  case might apply to a situation where the performance is deteriorating as that in model 4.

7. For any hazard rate function (human error rate)  $h(t)$ , the associated distribution and density functions can be obtained by integrating Equation (42) and using  $f(t) = \frac{dF(t)}{dt}$  to get the relationship

$$F(t) = 1 - \exp\left[-\int_0^t h(s) ds\right]$$

or

$$f(t) = h(t) \exp\left[-\int_0^t h(s) ds\right].$$

## References

1. L. J. Bain and M. Engelhardt, Introduction to Probability and Mathematical Statistics, (1991), 2nd edition, PWS-KENT Publishing Co., Boston.
2. B.S. Dhillon, Human Reliability: With Human Factors, (1986), Pergamon Press Inc., New York.
3. W. Feller, An Introduction to Probability Theory and Its Application, (1968), Vol. I, John Wiley & Sons, New York.
4. K. Inaba and R. Matson, Measurement of Human Errors With Existing Data, (1968), Proceedings of the Seventh Annual Reliability and Maintainability Conference, IEEE, New York, PP. 301-307.
5. K. S. Park, "Human Reliability With Probabilistic Learning in Discrete and Continuous Tasks: Conceptualization and Modeling," (1985), Microelectronics and Reliability, Vol. 25.
6. M. Parzen, Stochastic Processes, (1962), Holden-Day, Inc., San Francisco.







